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AdS/CFT duality for non-anticommutative supersymmetric gauge theory

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Abstract: We construct type IIB supergravity duals of non-anticommutative deformed $\mathcal{N} = 4$ SU(N) gauge theories. We consider in particular deformations preserving $\mathcal{N} = (1, 0)$ and $\mathcal{N} = (1/2, 0)$ supersymmetry. Such theories can be realised on N D3-branes in specific self-dual 5-form backgrounds. We show that the required 5-form field strengths can be produced by configurations of intersecting D3-branes and we are then able to construct the supergravity solutions in the near-horizon limit. We consider some consequences of this duality, in particular showing that the gravity duals predict that the dimensions of a subset of BPS operators are not modified by the deformation.

Keywords: AdS-CFT Correspondence, Superspaces, Intersecting branes models, Supersymmetric gauge theory.

Contents

1. Introduction

The AdS/CFT correspondence [1−4] is an explicit realization of the holographic principle [5, 6]. It is the best understood example of a gauge/gravity duality and offers promising opportunities to tackle and understand some of the most difficult problems in theoretical high energy physics such as the quantum properties of spacetime and the confinement problem in gauge theory.

Owing to different motivations, various generalizations of the dualities have been considered. We note in particular the ones [7, 8] for noncommutative supersymmetric Yang-Mills and [9] where the duality is characterized by a very interesting extended action of $\mathrm{SL}(2, \mathbb{Z})$ [11, 10]. Both of these deformations of the original AdS/CFT duality are motivated by having a deformed $*$-product on the field theory side. In the first case, the spacetime is deformed by a Moyal product induced by a constant NSNS B-field which lives on the worldvolume of the D3-branes, while in the second case, the product between fields carrying different $\mathrm{U}(1)$ charges is deformed due to a nontrivial twist in the TsT-transformation. In this regard, it is natural and of interest to construct a gauge/gravity duality for the non-anticommutative supersymmetric gauge theory [12, 13] where the fermionic coordinates of the superspace are deformed with a non-anticommutative $*$-product.[1] The goal of this paper is to construct the gauge/gravity duality for the non-anticommutative deformed $\mathcal{N} = 4$ supersymmetric Yang-Mills theory.

Non-anticommutative supersymmetric theories preserve a chiral fraction of the supersymmetries. That this is possible is because these theories are defined in Euclidean space and the left and right chiral sectors are not related by a complex conjugation. Due to their different supersymmetric structure, a priori these theories could have quite different quantum properties from their undeformed cousins. Although power counting non-renormalizable, nevertheless they are renormalizable [15, 16] basically because in a Feynman diagram computation, the Hermitian conjugate partners which would be needed to generate divergent counter terms are missing. Moreover due to the existence of a superspace formulation, non-renormalization theorems exist as usual. In addition to having a very interesting mechanism of supersymmetry breaking, non-anticommutative supersymmetric theories also possess interesting non-perturbative properties [17].

In the original Maldacena AdS/CFT correspondence, the amount of preserved supersymmetry is maximal. Since holography is believed to be a generic property of quantum gravity, it is interesting to understand how gauge/gravity duality works in a less or non-supersymmetric setting, especially when the supersymmetry is preserved in a non-standard manner. This is another motivation for our goal.

Non-anticommutativity in string theory [18 − 20] was first discovered by Ooguri and Vafa [18], who observed that a self-dual graviphoton field strength $C_{\mu\nu}$ induces a deformation in the fermionic part of the 4-dimensional superspace. Seiberg proposed another type of deformation which imposes commutativity in the chiral coordinates [12]. The deformation keeps $\mathcal{N} = 1/2$ supersymmetry in the case of simple supersymmetry, or more specifically, $\mathcal{N} = (1/2, 0)$ of the original $\mathcal{N} = (1/2, 1/2)$ supersymmetry [2]. The deformed superspace has algebra

$$\{\theta^\alpha, \theta^\beta\} = C^{\alpha\beta}, \tag{1.1}$$

$$\{\bar\theta^{\dot\alpha}, \bar\theta^{\dot\beta}\} = \{\theta^\alpha, \bar\theta^{\dot\beta}\} = 0, \tag{1.2}$$

$$[y^\mu, y^\nu] = [y^\mu, \theta^\alpha] = [y^\mu, \bar\theta^{\dot\alpha}] = 0, \tag{1.3}$$

where $y^\mu = x^\mu + i\theta^\alpha \sigma^\mu_{\alpha\dot\alpha} \bar\theta^{\dot\alpha}$. The deformation is described by the constant $C^{\alpha\beta} = C_{\mu\nu}(\sigma^{\mu\nu})^{\alpha\beta}$.

The generalization to the extended supersymmetry is immediate. For the $\mathcal{N} = 2 = (1, 1)$ case, the deformation generalizes to

$$\{\theta^{\alpha i}, \theta^{\beta j}\} = C^{\alpha\beta ij}, \quad i = 1, 2, \tag{1.4}$$

[1]Non-anticommutative $\mathcal{N} = 4$ super Yang-Mills theory can also be realized through deforming the constraint equations defined on the Euclidean superspace $\mathbb{R}^{4|16}$ [14].

[2]In Euclidean space, the Grassmannian-odd coordinates θ^α and $\bar\theta^{\dot\alpha}$ are not related by complex conjugation, therefore it is more convenient to denote the simple $\mathcal{N} = 1$ supersymmetry as $\mathcal{N} = (1/2, 1/2)$. For the more general extended case, one can have $\mathcal{N} = (n/2, m/2)$ supersymmetry where n, m are the number of left and right chiral spinorial supersymmetry generators.

with all the other (anti)commutative relations remaining undeformed. The deformation parameter $C^{\alpha\beta ij}$ obviously satisfies $C^{\alpha\beta ij} = C^{\beta\alpha ji}$. The deformation parameter can be decomposed into irreducible parts [21, 22]

$$C^{\alpha\beta ij} = \epsilon^{\alpha\beta}\epsilon^{ij} I + C^{(\alpha\beta)(ij)}. \tag{1.5}$$

The deformation described by the first term is called the singlet deformation. It preserves Euclidean SO(4) invariance and SU(2) R-symmetry, and breaks $\mathcal{N} = (1,1)$ supersymmetry down to $\mathcal{N} = (1,0)$. The singlet deformation can be obtained from string theory in a constant RR scalar background [23]. The deformation described by the second term in (1.5) is called the non-singlet deformation. It retains $\mathcal{N} = (1,0)$ supersymmetry for generic $C^{\alpha\beta ij}$. However for particular deformation parameters such that

$$C^{\alpha\beta ij} = C^{\alpha\beta} b^{ij} \tag{1.6}$$

and with $\det b = 0$, the preserved supersymmetry is enhanced to $\mathcal{N} = (1, 1/2)$ [21]. The non-singlet deformation can be obtained from string theory in a constant RR 5-form background [23 − 25]. $\mathcal{N} = 4$ lightcone superspace could be defined. However non-anticommutative deformations of it have not been considered.

The non-anticommutative deformation (1.6) can be obtained from string theory in a particular RR 5-form background of the form [24, 25]

$$F_{\mu\nu abc} = f_{\mu\nu} g_{abc}, \tag{1.7}$$

where $\mu, \nu = 0, 1, 2, 3$ denote the 4-dimensional indices and $a, b, c, = 4, \ldots, 9$ are the indices of the transverse space. This 5-form is self-dual both in the 4-spacetime directions and in the transverse 6 dimensions

$$f_{\mu\nu} = \frac{1}{2!}\epsilon_{\mu\nu\rho\sigma} f_{\rho\sigma}, \quad g_{abc} = \frac{-i}{3!}\epsilon_{abcdef} g_{def}. \tag{1.8}$$

In other words, the RR 5-form has the non-vanishing components $F_{\mu\nu abc}$ and satisfies the "double self-duality" condition

$$\begin{aligned} F_{\mu\nu abc} &= \frac{1}{2!}\epsilon_{\mu\nu\rho\lambda} F_{\rho\lambda abc}, \\ F_{\mu\nu abc} &= \frac{-i}{3!}\epsilon_{abcdef} F_{\mu\nu def}. \end{aligned} \tag{1.9}$$

Note that g_{abc} and hence the RR 5-form is necessarily complex since we are dealing with Euclidean signature.

The action for the non-anticommutative SYM theory can be obtained using the superspace construction. For the deformed $\mathcal{N} = (1/2, 1/2)$ superspace, see [12] for pure SYM and [26] for SYM theory with matter. For the deformed $\mathcal{N} = (1,1)$ case, one may use harmonic superspace. See [27, 28, 23, 29] for the case with singlet deformation, and [27, 30 − 32] for the non-singlet deformation. It can also be obtained from string theory as the worldvolume action of D3-branes. More specifically, the pure SYM action with $\mathcal{N} = (1/2, 0)$ or $\mathcal{N} = (1, 0)$ supersymmetry can be obtained as the worldvolume action of D3-branes in a

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orbifold with constant graviphoton background [33, 24]. Deformations of the $\mathcal{N}=4$ SYM action with $\mathcal{N}=(1/2,0)$ or $\mathcal{N}=(1,0)$ supersymmetry can be obtained as the worldvolume action of D3-branes in a specific configuration of RR 5-form flux [25]. In this paper we will be interested in constructing a gauge/gravity duality for the non-anticommutative deformed $\mathcal{N}=4$ SYM theories with $\mathcal{N}=(1/2,0)$ and $\mathcal{N}=(1,0)$ supersymmetry.

The paper is organized as follows. In section 2, we present the construction of the non-anticommutative deformed $\mathcal{N}=4$ SYM theories with $\mathcal{N}=(1/2,0)$ and $\mathcal{N}=(1,0)$ supersymmetry as the worldvolume action of D3-branes with a particular configuration of RR 5-form flux. In section 3, we realize the configurations as intersecting brane systems and obtain the supergravity duals of these non-anticommutative deformed $\mathcal{N}=4$ gauge theories. In section 4, we focus on the theory with $\mathcal{N}=(1,0)$ supersymmetry and analyse the duality. In particular we perform a standard bulk-to-boundary analysis to extract the two point correlation function for the field theory. We find that the deformation modifies only the overall coefficient, but leaves the form of the two-point function unchanged. This implies that there exists a sector of BPS operators whose dimensions are unmodified by the deformation.

2. The non-anticommutative deformations of the $\mathcal{N}=4$ SYM

2.1 D3-brane realization of $\mathcal{N}=4$ SYM

Consider N D3-branes in the 0123-directions. The Lorentz group SO(10) is decomposed into SO(4) $\times$ SO(6) and the spin fields can be decomposed as $(S^\alpha S^A, S_{\dot\alpha}S_A)$ and $(\tilde S^\beta \tilde S^B, \tilde S_{\dot\beta}\tilde S_B)$ where $S^\alpha, \tilde S^\alpha$ and $S_{\dot\alpha}, \tilde S_{\dot\alpha}$ $(\alpha, \dot\alpha = 1,2)$ are four dimensional Weyl spinors and $S_A, \tilde S_A$ and $S^A, \tilde S^A$ $(A=1,2,3,4)$ are six-dimensional Weyl spinors. The presence of a constant RR background can be described using the RR vertex operator. In the $(-\frac{1}{2},-\frac{1}{2})$ picture, the RR vertex operator takes the form

$$V_{\mathcal{F}} = (2\pi\alpha)^{3/2}\tilde S^T \mathcal{C}\mathcal{F} S e^{-\phi/2}e^{-\tilde\phi/2} \tag{2.1}$$

where $\mathcal{F} := \sum_p F_{\mu_1\cdots\mu_{p+1}}\Gamma^{\mu_1\cdots\mu_{p+1}}/p!$ and $\mathcal{C}$ is the charge conjugation matrix [3]. Decomposing the spinor indices with respect to SO(4) $\times$ SO(6), we have

$$V_{\mathcal{F}} = (2\pi\alpha)^{3/2}\mathcal{F}^{\alpha\beta AB} S_\alpha S_A \tilde S_\beta \tilde S_B e^{-\phi/2}e^{-\tilde\phi/2} + \cdots\,, \tag{2.3}$$

where $\cdots$ denotes contributions from the components of $\mathcal{F}$ other than $\mathcal{F}^{\alpha\beta AB}$. Due to their different tensor structure, these components will not be relevant for our discussion.

To obtain the tensor structure of the deformation relations (1.1) or (1.4), it is necessary to consider a configuration of RR fields such that the only non-vanishing components are

[3] $\mathcal{F}$ and $F_{\mu_1\cdots\mu_{p+1}}$ are of dimension $[L]^{-2}$ here. This is different from the normal dimension of $[L]^{-1}$ for the RR gauge field strength in supergravity theory

$$S = (2\kappa_{10}^2)^{-1}\int d^{10}x\sqrt{g}(R - \frac{1}{2}F^2). \tag{2.2}$$

the symmetric ones $\mathcal{F}^{(\alpha\beta)(AB)}$. Here $(\alpha\beta)$, (AB) represent symmetrization of the indices. This can be achieved by turning on the RR 5-form configuration (1.7), (1.8). As a result

$$\mathcal{F}^{\alpha\beta AB} = f_{\mu\nu} g_{abc} (\sigma^{\mu\nu})^{\alpha\beta} (\Sigma^{abc})^{AB}, \tag{2.4}$$

where

$$\sigma^{\mu\nu} := \frac{1}{4}(\sigma^\mu \bar{\sigma}^\nu - \sigma^\nu \bar{\sigma}^\mu), \quad \Sigma^{abc} := \Sigma^{[a}\overline{\Sigma}^b \Sigma^{c]} \tag{2.5}$$

and they are self-dual:

$$\sigma^{\mu\nu} = \frac{1}{2!}\epsilon^{\mu\nu\rho\lambda}\sigma^{\rho\lambda}, \quad \Sigma^{abc} = \frac{i}{3!}\epsilon^{abcdef}\Sigma^{def}, \tag{2.6}$$

in which σ_μ, $\bar{\sigma}_\mu$ and Σ^a, $\overline{\Sigma}^a$ are gamma matrices for the 4-dimensional and the 6-dimensional spaces; one example of the basis is given in appendix.

The quantization of the string worldsheet coupled to the RR-fields leads to the non-anticommutative relations (1.1) and (1.4) [18 – 20, 23], and one expects the D3-brane worldvolume action to possess supersymmetry that is carried by the deformed superspace. The SYM action on the worldvolume of the D3-branes in the presence of a constant RR 5-form flux $\mathcal{F}^{(\alpha\beta)(AB)}$ satisfying the double self-dual condition (1.9) was computed in [25] using string perturbation theory. The deformation is determined by the parameters

$$C^{\alpha\beta AB} := (2\pi\alpha')^{3/2}\mathcal{F}^{\alpha\beta AB}, \tag{2.7}$$

which are kept fixed in the $\alpha' \to 0$ limit. The action was computed up to the first order in $\mathcal{F}$. The worldvolume action possess $\mathcal{N} = (1/2, 0)$ supersymmetry when $\mathcal{F}^{\alpha\beta AB}$ is of rank one in the (A, B)-space [25]. When it is of rank two in the (A, B)-space, one expects the worldvolume action to have $\mathcal{N} = (1, 0)$ supersymmetry. An alternative way to construct the deformed supersymmetric action is to use deformed $\mathcal{N} = (1, 1)$ harmonic superspace.

2.2 Non-anticommutative SYM with $\mathcal{N} = (1, 0)$ supersymmetry

2.2.1 RR-flux configuration for $\mathcal{N} = (1, 0)$ supersymmetry

To introduce a deformation to the $\mathcal{N} = (1, 1)$ superspace, the RR-5 form $\mathcal{F}^{\alpha\beta AB}$ should be non-vanishing only for a 2×2 sub-block of the indices for A, B. This can be achieved with the following configuration of RR 5-form:

$$\begin{aligned} F_{01456} &= -iF_{01789} = F_{23456} = -iF_{23789} = c, \\ F_{01786} &= -iF_{01459} = F_{23786} = -iF_{23459} = c, \end{aligned} \tag{2.8}$$

where

$$c := F_{01456} \tag{2.9}$$

is a constant. The first or the second line of (2.8) is respectively a minimal configuration which satisfies (1.7), (1.8). By having this particular combination of these minimal configurations, $\mathcal{F}$ is given by

$$\begin{aligned} \mathcal{F}^{\alpha\beta AB} &= 24c(\sigma^{01} + \sigma^{23})^{\alpha\beta}(\Sigma^{456} + i\Sigma^{459})^{AB} \\ &= 24ic(\tau^3)^{\alpha\beta} M^{AB} \end{aligned} \tag{2.10}$$

and

$$M := \Sigma^{456} + i\Sigma^{459}. \tag{2.11}$$

To proceed further, one needs an explicit representation of the Σ-matrices. For example, one can identify the Σ-matrices here with the canonical choice of Σ-matrices (A.6) given in the appendix. For example if we take

$$\Sigma^{6,9,4,5,7,8} = \left(\Sigma^{4,5,6,7,8,9}\right)_{\text{appendix}}, \tag{2.12}$$

then

$$M = 2i \begin{pmatrix} \tau^1 & 0 \\ 0 & 0 \end{pmatrix} := M_0, \tag{2.13}$$

which is of rank 2. We remark that a different identification of the Σ-matrices gives a different M that is related to (2.13) by a bi-unitary transformation

$$M = \tilde{V} M_0 V, \tag{2.14}$$

where $\tilde{V} = V^T$, V are unitary. The transformation is bi-unitary since in general $\tilde{V}V \neq 1$. For example, for the identification

$$\Sigma^{6,9,4,5,7,8} = \left(\Sigma^{4,8,6,7,5,9}\right)_{\text{appendix}}, \tag{2.15}$$

we have

$$M = V^T \, 2i \begin{pmatrix} \tau^1 & 0 \\ 0 & 0 \end{pmatrix} V, \tag{2.16}$$

with

$$V = \frac{1}{\sqrt{2}} \begin{pmatrix} \tau^1 & -\tau^2 \\ -\tau^2 & \tau^1 \end{pmatrix}. \tag{2.17}$$

It is

$$V^T V = -V V^T = -i \begin{pmatrix} 0 & \tau^3 \\ \tau^3 & 0 \end{pmatrix} \neq 1 \tag{2.18}$$

and so the transformation (2.14) is not unitary, but a bi-unitary one. We note that the vertex operator with M given by (2.14) is equivalent to the one with $M = M_0$ under a change of basis for the spin field $S_A, \tilde{S}_A$

$$S \to VS, \quad \tilde{S} \to V\tilde{S}. \tag{2.19}$$

Thus we have shown that by turning on the constant RR 5-form field (2.8), $\mathcal{F}^{\alpha\beta AB}$ takes the factorized form (1.6) with $\det b \neq 0$, i.e. it is of rank 2 with respect to the A, B indices.

2.2.2 Harmonic superspace and non-anticommutative SYM

Given the non-anticommutative relation (1.4), the deformed $\mathcal{N} = 4$ action on the worldvolume of the D3-branes can be obtained as a supersymmetric action for the gauge superfield and hypermultiplet superfield of deformed harmonic superspace. Let us first give a brief

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introduction to harmonic superspace [35]. For a comprehensive review, we refer the reader to [36].

Let $(x^\mu, \theta_i^\alpha, \bar{\theta}^{\dot{\alpha}i})$ be the coordinates of the $\mathcal{N}=(1,1)$ superspace, where $\mu = 0,1,2,3$ are the spacetime indices, $\alpha, \dot{\alpha} = 1,2$ spinor indices and $i = 1,2$ are the indices of the SU(2) R-symmetry. The harmonic superspace is supplemented by the harmonic variables $u_i^\pm$ which form an SU(2) matrix:

$$\begin{aligned} u^{i+}u_i^- &= 1, \quad u^{+i}u_i^+ = u^{-i}u_i^- = 0, \\ \widetilde{u^{+i}} &= u_i^-. \end{aligned} \tag{2.20}$$

Here $\widetilde{\ }$ is the standard conjugation acting on the harmonic superspace. Its action on the other coordinates of the superspace is

$$\tilde{x}_A^\mu = x_A^\mu, \qquad \widetilde{\theta^{\pm\alpha}} = \varepsilon_{\alpha\beta}\theta^{\pm\beta}, \qquad \widetilde{\bar{\theta}^{\pm\dot{\alpha}}} = \varepsilon_{\dot{\alpha}\dot{\beta}}\bar{\theta}^{\pm\dot{\beta}}, \tag{2.21}$$

where

$$\begin{aligned} \theta_\alpha^\pm &:= u_i^\pm \theta_\alpha^i, \qquad\qquad \bar{\theta}_{\dot{\alpha}}^\pm := u_i^\pm \bar{\theta}_{\dot{\alpha}}^i, \\ x_A^\mu &= x^\mu - i(\theta^+\sigma^\mu\bar{\theta}^- + \theta^-\sigma^\mu\bar{\theta}^+) \end{aligned} \tag{2.22}$$

is the analytic basis of the harmonic superspace. And as a result we have the condition on the parameters deforming the $\mathcal{N}=(1,1)$ superspace:

$$\widetilde{C^{\alpha\beta ij}} = C_{\alpha\beta ij}. \tag{2.23}$$

Using these variables, one can introduce the harmonic projection of the supercovariant derivatives

$$D_\alpha^\pm = u_i^\pm D_\alpha^i, \quad \bar{D}_\alpha^\pm = u_i^\pm \bar{D}_\alpha^i. \tag{2.24}$$

Instead of chiral superfields, one considers analytic superfields in harmonic superspace. They satisfy $D_\alpha^+\Phi = \bar{D}_{\dot{\alpha}}^+\Phi = 0$. The supercharges and supercovariant derivatives take the form

$$\begin{aligned} Q_\alpha^+ &= \frac{\partial}{\partial\theta^{-\alpha}} - 2i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{+\dot{\alpha}}\frac{\partial}{\partial x_A^\mu}, & Q_\alpha^- &= -\frac{\partial}{\partial\theta^{+\alpha}} \\ \bar{Q}_{\dot{\alpha}}^+ &= \frac{\partial}{\partial\bar{\theta}^{-\dot{\alpha}}} + 2i\theta^{+\alpha}\sigma^\mu_{\alpha\dot{\alpha}}\frac{\partial}{\partial x_A^\mu}, & \bar{Q}_{\dot{\alpha}}^- &= -\frac{\partial}{\partial\bar{\theta}^{+\dot{\alpha}}}, \\ D_\alpha^+ &= \frac{\partial}{\partial\theta^{-\alpha}}, & D_\alpha^- &= -\frac{\partial}{\partial\theta^{+\alpha}} + 2i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{-\dot{\alpha}}\frac{\partial}{\partial x_A^\mu}, \\ \overline{D}_{\dot{\alpha}}^+ &= \frac{\partial}{\partial\bar{\theta}^{-\dot{\alpha}}}, & \overline{D}_{\dot{\alpha}}^- &= -\frac{\partial}{\partial\bar{\theta}^{+\dot{\alpha}}} - 2i\theta^{-\alpha}\sigma^\mu_{\alpha\dot{\alpha}}\frac{\partial}{\partial x_A^\mu}, \end{aligned} \tag{2.25}$$

in terms of which the condition for the analytic superfield can be solved easily and is of the form

$$\Phi = \Phi(x_A^\mu, \theta^+, \bar{\theta}^+, u). \tag{2.26}$$

One can expand the analytic superfield in θ and obtain a finite expansion with coefficients being functions of x_A and u. Each θ-component can be further expanded in terms of symmetrized products of u^+ and u^-. This second expansion is infinite and so each analytic superfield contains an infinite number of component fields.

It is convenient to introduce covariant derivatives with respect to $u^\pm$ compatible with the defining relations (2.20)

$$\begin{aligned} D^{++} &= u^{+i}\frac{\partial}{\partial u^{-i}}, \qquad\qquad D^{--} = u^{-i}\frac{\partial}{\partial u^{+i}}, \\ D^0 &= u^{+i}\frac{\partial}{\partial u^{+i}} - u^{-i}\frac{\partial}{\partial u^{-i}}. \end{aligned} \tag{2.27}$$

The operator D^0 measures the U(1) charges of the harmonics $u^\pm$. A function of charge q satisfies

$$D^0\phi^{(q)} = q\phi^{(q)} \tag{2.28}$$

and admits the expansion (here $q \geq 0$; for $q < 0$ there is an analogous formula)

$$\phi^{(q)}(u) = \sum_{n=0}^{\infty} \phi^{(i_1\cdots i_{q+n}j_1\cdots j_n)}\, u^+_{(i_1}\cdots u^+_{i_{q+n}}u^-_{j_1}\cdots u^-_{j_n)}. \tag{2.29}$$

The coefficients $\phi^{i_1\cdots j_m}$ are irreducible SU(2) tensors with isospin $(n+m)/2$.

The non-anticommutative $\mathcal{N} = (1,1)$ superspace has the deformed relation

$$\{\theta^{\alpha i}, \theta^{\beta j}\} = C^{\alpha\beta ij}, \quad i = 1,2. \tag{2.30}$$

The deformation is equivalent to a $*$-product

$$(f * g)(\theta) = f(\theta)\exp\left(-\frac{1}{2}\frac{\overleftarrow{\partial}}{\partial\theta^\alpha_i}\, C^{\alpha\beta}_{ij}\,\frac{\overrightarrow{\partial}}{\partial\theta^\beta_j}\right)g(\theta). \tag{2.31}$$

In the harmonic superspace approach, the $\mathcal{N} = 2$ gauge multiplet is described by a charge 2 analytic superfield $V^{++} = V^{++M}T^M$ where T^M are the Lie algebra generators. We will consider U(N) in this paper. Under the deformed U(N) gauge group, the gauge multiplet transforms as

$$\delta_\Lambda V^{++} = -D^{++}\Lambda + i[V^{++} \stackrel{*}{,} \Lambda] \tag{2.32}$$

where Λ is an analytic superfield parameter. The action of $\mathcal{N} = 2$ SYM is given by [37, 21]

$$S_V = \frac{1}{2}\sum_{n=2}^{\infty}\frac{(-i)^n}{n}\mathrm{tr}\int d^{12}z du_1\cdots du_n \frac{V^{++}(z,u_1) * V^{++}(z,u_2) * \cdots * V^{++}(z,u_n)}{(u_1^+u_2^+)(u_2^+u_3^+)\cdots(u_n^+u_1^+)}, \tag{2.33}$$

where $z = (x, \theta^\alpha_i, \bar\theta^{\dot\alpha}_i)$.

As for the hypermultiplet, it can be described either by a complex analytic superfield q^+ with U(1) charge $+1$ or by a real analytic neutral superfield ω. These descriptions are known to be related to each other via a duality [36] and one can restrict to either description. Similar consideration applies in the deformed case. To construct the deformed $\mathcal{N} = 4$ SYM, let us consider q^+ in the adjoint representation. The coupling of q^+ to the $\mathcal{N} = 2$ gauge multiplet is given by

$$S_q = -\int d\zeta du \mathrm{Tr}\, \bar{q}^+ * (D^{++} + i[V^{++} \stackrel{*}{,} q^+]). \tag{2.34}$$

where

$$d\zeta := d^4 x_A d^4 \theta^- . \tag{2.35}$$

The $\mathcal{N}=4$ SYM theory can be written down in terms of these $\mathcal{N}=2$ superfields. In fact, by using an $\mathcal{N}=2$ gauge multiplet and an $\mathcal{N}=2$ hypermultiplet q^+ in the adjoint representation, the $\mathcal{N}=4$ action can be written as

$$S_{\mathrm{SYM}} = S_V + S_q . \tag{2.36}$$

For a generic non-singlet deformation (1.6) with generic b such that $\det b \neq 0$, the theory has $\mathcal{N}=(1,0)$ supersymmetry.

The action (2.36) is written in terms of $\mathcal{N}=(1,1)$ superfields. To rewrite it in terms of component fields, one needs to substitute the component expansion of the superfield and carry out the integrals in θ and u. As we remarked above, the component expansion of an analytic superfield contains an infinite number of component fields and auxiliary fields. However many of these can be gauged away. For example, one can utilize the infinite degrees of freedom present in the analytic gauge parameter Λ to eliminate all the auxiliary fields in the gauge superfield V^{++}. In the Wess-Zumino gauge, V^{++}_{WZ} has only a finite number of physical components [21]. For the hypermultiplet superfield q^+, the auxiliary fields can be eliminated from the action using the classical equation of motion for q^+. We refer the reader to [21] for the case of a $\mathrm{U}(1)$ gauge group. The generalization to $\mathrm{U}(N)$ is straightforward.

Although the resulting component action is manifest in supersymmetry, the gauge transformations of the component fields are typically obscured and become non-canonical. This was first observed in [12] for the deformed $\mathcal{N}=(1/2,1/2)$ superspace. To obtain component fields which have canonical gauge transformations, one must perform a field redefinition. The redefined component fields admit canonical gauge transformations, but their supersymmetry transformations are deformed. This can be worked out explicitly and fully in the deformed $\mathcal{N}=(1/2,1/2)$ case. However this becomes much more complicated for the deformed $\mathcal{N}=(1,1)$ case [27, 28, 23, 30]. Both the field redefinition and the deformed supersymmetry transformations involve infinite series expansions in the deformation parameter. Therefore although there is in principle no difficulty to write down the deformed action explicitly, the procedure is rather involved and we will not carry out its evaluation here.

2.3 Non-anticommutative SYM with $\mathcal{N}=(1/2,0)$ supersymmetry

To construct a deformation of the $\mathcal{N}=4$ SYM theory with $\mathcal{N}=(1/2,0)$ supersymmetry, one can first write the $\mathcal{N}=4$ theory in terms of $\mathcal{N}=(1/2,1/2)$ superfields and then introduces non-anticommutative deformation to the $\mathcal{N}=(1/2,1/2)$ superspace. In this case, the RR-5 form $\mathcal{F}^{\alpha\beta AB}$ should be non-vanishing only for a 1×1 sub-block of the indices for A, B.

This can be achieved by turning on further components in addition to those in (2.8) which has the effect of further reducing the rank of $\mathcal{F}^{\alpha\beta AB}$. Up to equivalence, the appropriate RR 5-form configuration is

$$\begin{array}{llllll} F_{01456} &= -iF_{01789} &= F_{23456} &= -iF_{23789} &= & c, \\ F_{01786} &= -iF_{01459} &= F_{23786} &= -iF_{23459} &= & c, \\ F_{01476} &= \ \ iF_{01589} &= F_{23476} &= \ \ iF_{23589} &= & ic, \\ F_{01586} &= \ \ iF_{01479} &= F_{23586} &= \ \ iF_{23479} &= & -ic, \end{array} \tag{2.37}$$

where

$$c := F_{01456} \tag{2.38}$$

is a constant. The matrix $\mathcal{F}$ takes the form

$$\mathcal{F}^{\alpha\beta AB} = 24ic(\tau^3)^{\alpha\beta} M^{AB}, \tag{2.39}$$

where in this case M^{AB} is given by

$$M := \Sigma^{456} + i\Sigma^{459} + i(\Sigma^{476} + i\Sigma^{479}). \tag{2.40}$$

Without loss of generality, we take the same identification of Σ-matrices as in (2.12) to obtain

$$M = 4iU^T M_0 U, \tag{2.41}$$

where

$$M_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{and} \quad U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{2.42}$$

As before, the presence of U is a matter of choice of basis and can be absorbed by a transformation of the spin fields. Therefore the configuration (2.37) of the RR 5-form flux gives rise to a deformation that is governed by (2.39) with M of rank 1, and corresponds to a non-anticommutative deformation of the $\mathcal{N} = (1/2, 1/2)$ superspace.

As mentioned above, the non-anticommutative deformed SYM theory can be obtained easily using the deformed $\mathcal{N} = 1$ superspace. The theory admits $\mathcal{N} = (1/2, 0)$ supersymmetry. It is interesting to note that the additional terms in the action which deform the theory have an interpretation as the Chern-Simons couplings of the D3-brane to a certain constant RR 5-form background [38].

3. The supergravity solution

It is easy to check that the constant RR 5-form field strength (2.8) does not generate any energy-momentum tensor in flat Euclidean space:

$$T_{MN} = F_{MM_1M_2M_3M_4} F_N{}^{M_1M_2M_3M_4} - \frac{1}{10} g_{MN} F^2 = 0. \tag{3.1}$$

However this is no longer the case once one takes into account the backreaction of the N D3-branes, which turns the flat spacetime to $AdS_5 \times S^5$. Our goal now is to construct the supergravity solution which would give rise to the components (2.8) for the RR 5-form field on the worldvolume of the N D3-branes. Moreover, as a deformation, the solution should reduce back to the original $AdS_5 \times S^5$ background when the deformation is turned off.

3.1 Supergravity dual for the $\mathcal{N} = (1,0)$ case

In order to obtain the desired configuration (2.8) of the RR 5-form flux, we consider the following configuration of intersecting D3-branes,

$$\begin{array}{c|lcccccccccl} D3_1 & (& 0 & 1 & 2 & 3 & & & & & &) \\ D3_2 & (& 0 & 1 & & & 4 & 5 & & & &) \\ D3_{2'} & (& 0 & 1 & & & & & & 7 & 8 &) \\ D3_3 & (& & & 2 & 3 & 4 & 5 & & & &) \\ D3_{3'} & (& & & 2 & 3 & & & & 7 & 8 &) \end{array} . \tag{3.2}$$

Here $D3_1$ denotes the original N D3-branes; and we have introduced four additional sets of D3-branes. Let us check supersymmetry. In type IIB string theory, the two supersymmetries $\varepsilon_1, \varepsilon_2$ are of the same chirality. The set of N D3-branes imposes the condition

$$\Gamma^{0123}\varepsilon_1 = \varepsilon_2. \tag{3.3}$$

This condition relates the two supersymmetries and hence reduces the supersymmetry by one half. Now introduce the other 4 sets of branes $D3_2, D3_{2'}$, $D3_3, D3_{3'}$. This imposes additionally the conditions

$$\Gamma^{0145}\varepsilon_1 = \varepsilon_2, \quad \Gamma^{0178}\varepsilon_1 = \varepsilon_2, \quad \Gamma^{2345}\varepsilon_1 = \varepsilon_2, \quad \Gamma^{2378}\varepsilon_1 = \varepsilon_2. \tag{3.4}$$

The 4 conditions in (3.4) are not all independent. In fact there are only 3 independent equations in (3.4) and (3.4) is equivalent to the following system:

$$\Gamma^{2378}\varepsilon_1 = \varepsilon_2, \quad \Gamma^{0123}\varepsilon_1 = -\varepsilon_1, \quad \Gamma^{4578}\varepsilon_1 = -\varepsilon_1. \tag{3.5}$$

Together with (3.3), we see that generically our set of intersecting branes preserves 1/16 of the type IIB supersymmetry, i.e. 2 supersymmetries are preserved. However in the near horizon limit of the N D3-branes, the condition (3.3) is lifted and all the 32 supersymmetries are preserved. Therefore, in this limit, we only have the conditions (3.5). The first of these conditions gives ε_2 once ε_1 is solved. The second and the third conditions in (3.5) impose 2 conditions on ε_1 which means 4 supersymmetries are preserved. Moreover the 4 supersymmetries are chiral both in the 4-dimensional and in the 6-dimensional sense. Hence we can denote the preserved supersymmetries by

$$\varepsilon^{\alpha A}, \qquad \alpha = 1,2; \quad A = 1,2. \tag{3.6}$$

This matches precisely with the preserved $\mathcal{N} = (1,0)$ supersymmetries in the non-anti-commutative SYM theory.

The metric of our intersecting branes system is given by

$$\begin{aligned} ds^2 = &\sqrt{\frac{H_3 H_{3'}}{H_1 H_2 H_{2'}}}(dx_0^2 + dx_1^2) + \sqrt{\frac{H_2 H_{2'}}{H_1 H_3 H_{3'}}}(dx_2^2 + dx_3^2) + \sqrt{\frac{H_1 H_{2'} H_{3'}}{H_2 H_3}}(dx_4^2 + dx_5^2) \\ &+ \sqrt{\frac{H_1 H_2 H_3}{H_{2'} H_{3'}}}(dx_7^2 + dx_8^2) + \sqrt{H_1 H_2 H_3 H_{2'} H_{3'}}(dx_6^2 + dx_9^2) \end{aligned} \tag{3.7}$$

and the RR 5-form is

$$F = F_0 + F_1, \tag{3.8}$$

where

$$F_0 := d\left(\frac{1}{H_1}\right)dx^{0123} + \text{dual}, \tag{3.9}$$

$$F_1 := d\left(\frac{1}{H_2}\right)dx^{0145} + d\left(\frac{1}{H_{2'}}\right)dx^{0178} + d\left(\frac{1}{H_3}\right)dx^{2345} + d\left(\frac{1}{H_{3'}}\right)dx^{2378} + \text{dual}. \tag{3.10}$$

F_0 is the RR 5-form sourced by the original set of N D3-branes, and F_1 is sourced by the additional sets of branes.

In order that no components of the RR 5-form other than those that are present in (2.8) are activated, we choose the harmonic functions $H_2, H_{2'}, H_3, H_{3'}$ to be functions of x_6 and x_9 only. Moreover to produce the complex structure of the RR 5-form in the equation (2.8), it is necessary that H_2 and $H_{2'}$ depend on x_6, x_9 in a particular way:

$$H_2 = H_2(z), \quad H_{2'} = H_{2'}(z), \quad H_3 = H_3(z), \quad H_{3'} = H_{3'}(z) \tag{3.11}$$

where

$$z = x_6 + ix_9 \tag{3.12}$$

is a complex variable. In other word, the branes $D3_2, D3_{2'}, D3_3, D3_{3'}$ are smeared and have effectively a single transverse direction.

The equations of motion for this system of partially localised intersecting branes are given by the curved space Laplace equations [39]

$$(H_2H_3\partial_i^2 + H_{2'}H_{3'}\partial_m^2 + \partial_a^2)H_1 = 0, \tag{3.13}$$

$$\partial_a\left(\frac{H_2^2}{H_{3'}^2}\partial_a\left(\frac{1}{H_2}\right)\right) = 0, \tag{3.14}$$

$$\partial_a\left(\frac{H_{2'}^2}{H_3^2}\partial_a\left(\frac{1}{H_{2'}}\right)\right) = 0, \tag{3.15}$$

$$\partial_a\left(\frac{H_3^2}{H_{2'}^2}\partial_a\left(\frac{1}{H_3}\right)\right) = 0, \tag{3.16}$$

$$\partial_a\left(\frac{H_{3'}^2}{H_2^2}\partial_a\left(\frac{1}{H_{3'}}\right)\right) = 0, \tag{3.17}$$

where we have used $i = 4, 5$ to denote the indices in the x_4, x_5 directions and $\partial_i^2 := \partial_4^2 + \partial_5^2$ is the 2-dimensional flat Laplacian. Similarly $\partial_m^2 := \partial_7^2 + \partial_8^2$ and $\partial_a^2 := \partial_6^2 + \partial_9^2$. Due to (3.11), the equations (3.14)–(3.17) are satisfied immediately. Since the branes $D3_2, D3_{2'}, D3_3, D3_{3'}$ are smeared and have effectively a single transverse direction, the charge associated with them is well defined only if F_1 as given by (3.10) is well-defined at $|z| = \infty$. Moreover we would like to reproduce the components of the RR flux precisely at the worldvolume of the set of N D3-branes, i.e. at $x_4 = x_5 = x_6 = x_7 = x_8 = x_9 = 0$. We obtain the unique solution

$$H_2 = H_{2'} = H_3 = H_{3'} = \frac{1}{1 + cz}. \tag{3.18}$$

In this case, the RR 5-form field strength (3.10) is actually constant and equal to (2.8) everywhere.

Finally the equation for H_1 reduces to

$$\left(\partial_i^2 + \partial_m^2 + \frac{1}{H_2^2}\partial_a^2\right) H_1 = 0. \tag{3.19}$$

Naively one may try to treat c as a small parameter and solve (3.19) by solving the differential equation perturbatively. This is messy however. A much simpler way to solve (3.19) in closed form is due to the following observation. We first rewrite the Laplacian $\partial_a^2 = 4\partial_z\partial_{\bar{z}}$ where $\bar{z} := x_6 - ix_9$ and introduce

$$w := \int H_2^2 dz = \frac{z}{1+cz}, \tag{3.20}$$

where we have chosen the integration constant such that $w = 0$ when $z = 0$. In terms of w and $\bar{z}$, the equation (3.19) can be rewritten as

$$(\partial_i^2 + \partial_m^2 + 4\partial_w\partial_{\bar{z}})H_1 = 0. \tag{3.21}$$

Except for the fact that w is not the complex conjugate of $\bar{z}$, this is formally the same Laplace equation as in the undeformed $AdS_5 \times S^5$ case. This fact allows us to solve (3.21) easily. We obtain

$$H_1 = 1 + \frac{R^4}{\rho^4}, \tag{3.22}$$

where

$$\frac{R^4}{\alpha'^2} := 4\pi g_s N = \lambda \tag{3.23}$$

and

$$\rho^2 := x_i^2 + x_m^2 + w\bar{z} \tag{3.24}$$

It is

$$\rho^2 = x_i^2 + x_m^2 + \frac{z\bar{z}}{1+cz} = x_i^2 + x_m^2 + \frac{w\bar{w}}{1-\bar{c}\bar{w}} \tag{3.25}$$

Formally the solution H_1 takes the same form as the undeformed one. That this is true is because the differential algebra involved does not care about the complex structure. Using the above results, the metric becomes

$$ds^2 = \frac{1}{\sqrt{H_1}}(dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2) + \sqrt{H_1}\left(dx_4^2 + dx_5^2 + dx_7^2 + dx_8^2 + \frac{dzd\bar{z}}{(1+cz)^2}\right). \tag{3.26}$$

It is clear that the singularity at $z = -1/c$ is infinitely far away and so the supergravity background is regular. We also remark that although the metric is invariant under SO(4) rotations in x^μ, $\mu = 0, 1, 2, 3$, the Euclidean Lorentzian symmetry SO(4) is broken by the RR 5-form. This agrees with the field theory.

Summarizing, our proposal is that the non-anticommutative SYM theory with deformation parameter (2.7), (2.8) is dual to the near horizon limit of the supergravity background given by the intersecting brane system (3.2). The near horizon limit is taken with

$$\tilde{x}^a := x^a/\alpha' \tag{3.27}$$

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fixed in the $\alpha' \to 0$ limit. Introducing

$$U := \rho/\alpha' \tag{3.28}$$

and also scaling c such that

$$\tilde{c} := \alpha' c = \alpha' F_{01456} \tag{3.29}$$

is fixed in the $\alpha' \to 0$ limit, we obtain the near horizon metric

$$\frac{ds^2}{\alpha'} = \frac{U^2}{\sqrt{\lambda}} dx_\mu^2 + \frac{\sqrt{\lambda}}{U^2}\left(d\tilde{x}_4^2 + d\tilde{x}_5^2 + d\tilde{x}_7^2 + d\tilde{x}_8^2 + \frac{d\tilde{z}d\bar{\tilde{z}}}{(1+\tilde{c}\tilde{z})^2}\right), \tag{3.30}$$

where $\tilde{z} := z/\alpha'$, and

$$U^2 = \tilde{x}_i^2 + \tilde{x}_m^2 + \frac{\tilde{z}\bar{\tilde{z}}}{1+\tilde{c}\tilde{z}}. \tag{3.31}$$

The RR 5-form is

$$F = F_0 + F_1, \tag{3.32}$$

where

$$\frac{F_0}{\alpha'^2} = d\left(\frac{U^4}{\lambda}\right) dx^{0123} + \text{dual} \tag{3.33}$$

and

$$\begin{aligned}\frac{F_1}{\alpha'^2} = \; & \tilde{c}(dx^0 dx^1 d\tilde{x}^4 d\tilde{x}^5 d\tilde{x}^6 + i dx^0 dx^1 d\tilde{x}^7 d\tilde{x}^8 d\tilde{x}^9 + dx^2 dx^3 d\tilde{x}^4 d\tilde{x}^5 d\tilde{x}^6 + i dx^2 dx^3 d\tilde{x}^7 d\tilde{x}^8 d\tilde{x}^9 \\ & + i dx^0 dx^1 d\tilde{x}^4 d\tilde{x}^5 d\tilde{x}^9 + dx^0 dx^1 d\tilde{x}^7 d\tilde{x}^8 d\tilde{x}^6 + i dx^2 dx^3 d\tilde{x}^4 d\tilde{x}^5 d\tilde{x}^9 + dx^2 dx^3 d\tilde{x}^7 d\tilde{x}^8 d\tilde{x}^6) \\ & + \text{dual}.\end{aligned} \tag{3.34}$$

Note that F_1 is well defined in the same limit (3.29) where the metric has a well-defined limit. Note also that the conditions (3.29) and (2.7) are indeed the same due to different normalization.

We remark that the RR-flux is necessarily complex in order to generate the non-anticommutative deformation. This is also reflected in the complexification of the metric through (3.35). We also remark that the effect of turning on the deformation (2.8) in the gauge theory is a simple replacement

$$dz d\bar{z} \to dw d\bar{z} \tag{3.35}$$

in the metric of the supergravity dual. It is remarkable that the effects of non-anti-commutativity can be summarized nicely in such a compact form through a simple change of variables. We will further comment on this in the discussion section.

3.2 Supergravity dual for the $\mathcal{N} = (1/2, 0)$ case

To obtain the configuration (2.37) of the RR 5-form flux, we consider the following configuration of intersecting D3-branes,

$$\begin{array}{r|l}
D3_1 & (\ 0\ 1\ 2\ 3\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ ) \\
D3_2 & (\ 0\ 1\ \ \ \ \ \ \ 4\ 5\ \ \ \ \ \ \ \ ) \\
D3_{2'} & (\ 0\ 1\ \ \ \ \ \ \ \ \ \ \ \ \ 7\ 8\ ) \\
D3_3 & (\ \ \ \ \ \ \ 2\ 3\ 4\ 5\ \ \ \ \ \ \ \ ) \\
D3_{3'} & (\ \ \ \ \ \ \ 2\ 3\ \ \ \ \ \ \ 7\ 8\ ) \\
D3_4 & (\ 0\ 1\ \ \ \ \ \ \ 4\ \ \ \ 7\ \ \ \ ) \\
D3_{4'} & (\ 0\ 1\ \ \ \ \ \ \ \ \ \ 5\ \ \ \ 8\ ) \\
D3_5 & (\ \ \ \ \ \ \ 2\ 3\ 4\ \ \ \ 7\ \ \ \ ) \\
D3_{5'} & (\ \ \ \ \ \ \ 2\ 3\ \ \ \ 5\ \ \ \ 8\ )
\end{array} . \tag{3.36}$$

Here $D3_1$ denotes the original N D3-branes; and we have introduced eight additional sets of D3-branes. The checking of supersymmetry is similar as before. We have from the original N D3-branes the condition

$$\Gamma^{0123}\varepsilon_1 = \varepsilon_2, \tag{3.37}$$

and the conditions

$$\Gamma^{0145}\varepsilon_1 = \varepsilon_2, \quad \Gamma^{0178}\varepsilon_1 = \varepsilon_2, \quad \Gamma^{2345}\varepsilon_1 = \varepsilon_2, \quad \Gamma^{2378}\varepsilon_1 = \varepsilon_2, \tag{3.38}$$

$$\Gamma^{0147}\varepsilon_1 = \varepsilon_2, \quad \Gamma^{0185}\varepsilon_1 = \varepsilon_2, \quad \Gamma^{2347}\varepsilon_1 = \varepsilon_2, \quad \Gamma^{2385}\varepsilon_1 = \varepsilon_2, \tag{3.39}$$

from the additional sets of branes. In the near horizon limit, this is equivalent to

$$\Gamma^{2378}\varepsilon_1 = \varepsilon_2, \quad \Gamma^{0123}\varepsilon_1 = -\varepsilon_1, \quad \Gamma^{4578}\varepsilon_1 = -\varepsilon_1 \tag{3.40}$$

and

$$\Gamma^{48}\varepsilon_1 = \varepsilon_1. \tag{3.41}$$

The presence of the additional projection condition (3.41) reduces further the unbroken supersymmetry to a single two-component chiral spinor and we can denote it as

$$\varepsilon^\alpha, \qquad \alpha = 1, 2. \tag{3.42}$$

Therefore, in the near horizon limit, the intersecting branes configuration preserves 1/16 of the type IIB supersymmetry, i.e. 2 supersymmetries. This matches precisely with the $\mathcal{N} = (1/2, 0)$ supersymmetries in the non-anticommutative SYM theory.

The metric of our intersecting branes system is given by

$$\begin{aligned}
ds^2 = {} & \sqrt{\frac{H_3 H_{3'} H_5 H_{5'}}{H_1 H_2 H_{2'} H_4 H_{4'}}}(dx_0^2 + dx_1^2) + \sqrt{\frac{H_2 H_{2'} H_4 H_{4'}}{H_1 H_3 H_{3'} H_5 H_{5'}}}(dx_2^2 + dx_3^2) \\
& + \sqrt{\frac{H_1 H_{2'} H_{3'} H_{4'} H_{5'}}{H_2 H_3 H_4 H_5}} dx_4^2 + \sqrt{\frac{H_1 H_{2'} H_{3'} H_4 H_5}{H_2 H_3 H_{4'} H_{5'}}} dx_5^2 \\
& + \sqrt{\frac{H_1 H_2 H_3 H_{4'} H_{5'}}{H_{2'} H_{3'} H_4 H_5}} dx_7^2 + \sqrt{\frac{H_1 H_2 H_3 H_4 H_5}{H_{2'} H_{3'} H_{4'} H_{5'}}} dx_8^2 \\
& + \sqrt{H_1 H_2 H_3 H_4 H_5 H_{2'} H_{3'} H_{4'} H_{5'}}(dx_6^2 + dx_9^2)
\end{aligned} \tag{3.43}$$

and the RR 5-form is

$$F = F_0 + F_1, \tag{3.44}$$

where

$$F_0 := d\left(\frac{1}{H_1}\right)dx^{0123} + \text{dual}, \tag{3.45}$$

$$\begin{aligned} F_1 := {} & d\left(\frac{1}{H_2}\right)dx^{0145} + d\left(\frac{1}{H_{2'}}\right)dx^{0178} + d\left(\frac{1}{H_3}\right)dx^{2345} + d\left(\frac{1}{H_{3'}}\right)dx^{2378} \\ & + d\left(\frac{1}{H_4}\right)dx^{0147} + d\left(\frac{1}{H_{4'}}\right)dx^{0158} + d\left(\frac{1}{H_5}\right)dx^{2347} + d\left(\frac{1}{H_{5'}}\right)dx^{2358} + \text{dual}. \end{aligned} \tag{3.46}$$

F_0 is the RR 5-form sourced by the original set of N D3-branes, and F_1 is sourced by the additional sets of branes. As before, we need to choose the functions H_2, $H_{2'}$, H_3, $H_{3'}$, $H_4, H_{4'}, H_5, H_{5'}$ to be functions of $z = x_6 + ix_9$ only.

The equations of motion for the system are:

$$\Big(H_2H_3(H_4H_5\partial_4^2 + H_{4'}H_{5'}\partial_5^2) + H_{2'}H_{3'}(H_4H_5\partial_7^2 + H_{4'}H_{5'}\partial_8^2) + \partial_a^2\Big)H_1 = 0, \tag{3.47}$$

$$\partial_a\left(\frac{H_4H_{4'}}{H_5H_{5'}}\frac{H_2^2}{H_{3'}^2}\partial_a\left(\frac{1}{H_2}\right)\right) = 0, \tag{3.48}$$

$$\partial_a\left(\frac{H_4H_{4'}}{H_5H_{5'}}\frac{H_{2'}^2}{H_3^2}\partial_a\left(\frac{1}{H_{2'}}\right)\right) = 0, \tag{3.49}$$

$$\partial_a\left(\frac{H_5H_{5'}}{H_4H_{4'}}\frac{H_3^2}{H_{2'}^2}\partial_a\left(\frac{1}{H_3}\right)\right) = 0, \tag{3.50}$$

$$\partial_a\left(\frac{H_5H_{5'}}{H_4H_{4'}}\frac{H_{3'}^2}{H_2^2}\partial_a\left(\frac{1}{H_{3'}}\right)\right) = 0, \tag{3.51}$$

$$\partial_a\left(\frac{H_2H_{2'}}{H_3H_{3'}}\frac{H_4^2}{H_{5'}^2}\partial_a\left(\frac{1}{H_4}\right)\right) = 0, \tag{3.52}$$

$$\partial_a\left(\frac{H_2H_{2'}}{H_3H_{3'}}\frac{H_{4'}^2}{H_5^2}\partial_a\left(\frac{1}{H_{4'}}\right)\right) = 0, \tag{3.53}$$

$$\partial_a\left(\frac{H_3H_{3'}}{H_2H_{2'}}\frac{H_5^2}{H_{4'}^2}\partial_a\left(\frac{1}{H_5}\right)\right) = 0, \tag{3.54}$$

$$\partial_a\left(\frac{H_3H_{3'}}{H_2H_{2'}}\frac{H_{5'}^2}{H_4^2}\partial_a\left(\frac{1}{H_{5'}}\right)\right) = 0, \tag{3.55}$$

where $a = 6, 9$. The equations (3.48) - (3.55) are satisfied immediately. As before, since the additional sets of branes are smeared and have effectively a single transverse direction, the charge associated with them is well defined only if F_1 as given by (3.46) is well-defined at $|z| = \infty$. Moreover we want to reproduce the components (2.37) of the RR flux precisely at the worldvolume of the set of N D3-branes. We obtain the unique solution

$$\begin{aligned} H_2 &= H_{2'} = H_3 = H_{3'} = \frac{1}{1+cz}, \\ H_4 &= H_5 = \frac{1}{1+icz} \\ H_{4'} &= H_{5'} = \frac{1}{1-icz}, \end{aligned} \tag{3.56}$$

The RR 5-form field strength (3.46) is constant and equal to (2.37) everywhere.

Finally, the equation for H_1 reduces to

$$\left(A\partial_i^2 + \frac{1}{A}\partial_m^2 + \frac{1}{H_2^2 H_4 H_{4'}}\partial_a^2\right) H_1 = 0, \tag{3.57}$$

where $i = 4, 7$, $m = 5, 8$, $a = 6, 9$ here, and the function A is defined by

$$A := \frac{H_4}{H_{4'}} = \frac{1 - icz}{1 + icz}. \tag{3.58}$$

The differential equation (3.57) can be solved as follows. Introduce the change of variable

$$w := \int H_2^2 H_4 H_{4'} dz = \frac{z}{2(1 + cz)} + \frac{1}{2c}\ln(1 + cz) - \frac{1}{4c}\ln(1 + c^2 z^2), \tag{3.59}$$

the equation (3.57) for H_1 becomes

$$\left(A\partial_i^2 + \frac{1}{A}\partial_m^2 + 4\partial_w \partial_{\bar{z}}\right) H_1 = 0. \tag{3.60}$$

This can be solved with the ansatz

$$H_1 = 1 + \frac{R^4}{\rho^4}, \tag{3.61}$$

where

$$\rho^2 = B_1(w) x_i^2 + B_2(w) x_m^2 + C(w)\bar{z}. \tag{3.62}$$

The equation (3.60) is satisfied if the following conditions on the coefficient functions $B_1(w), B_2(w)$ and $C(w)$ hold:

$$C' - 3C\frac{B_1'}{B_1} + \frac{B_2}{A} - 2AB_1 = 0, \tag{3.63}$$

$$C' - 3C\frac{B_2'}{B_2} + B_1 A - 2\frac{B_2}{A} = 0, \tag{3.64}$$

$$C' = \frac{1}{2}\left(B_1 A + \frac{B_2}{A}\right), \tag{3.65}$$

where $'$ here refers to differentiation with respect to w. It is easy to obtain from these equations

$$(B_1 B_2)' = 0, \tag{3.66}$$

$$\left(\frac{C}{B_1}\right)' = A, \tag{3.67}$$

$$\left(\frac{C}{B_2}\right)' = \frac{1}{A}. \tag{3.68}$$

By rescaling ρ, we can set the integration constant of (3.66) to be 1 and we obtain

$$B_1 = 1/B_2. \tag{3.69}$$

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The equations (3.67) and (3.68) then give

$$CB_1 = \int \frac{1}{A} dw, \quad \frac{C}{B_1} = \int A dw, \tag{3.70}$$

and hence

$$B_1(w(z)) = \frac{1}{B_2(w(z))} = \sqrt{\frac{N(z)}{D(z)}} \tag{3.71}$$

$$C(w(z)) = \sqrt{N(z)D(z)} \tag{3.72}$$

where

$$N(z) := \int \frac{1}{A} dw, \quad D(z) := \int A dw. \tag{3.73}$$

Substituting the definition (3.58) for $A(z)$ and recalling (3.59), we have

$$N(z) = \frac{1}{4c}\left[(1-i)\ln\frac{(1+cz)^2}{1+c^2z^2} + 2(1+i)\tan^{-1}(cz) - 2(1+i)\frac{c^2z^2}{(1+cz)(i+cz)}\right], \tag{3.74}$$

$$D(z) = \frac{1}{4c}\left[(1+i)\ln\frac{(1+cz)^2}{1+c^2z^2} + 2(1-i)\tan^{-1}(cz) - 2(1-i)\frac{c^2z^2}{(1+cz)(-i+cz)}\right]. \tag{3.75}$$

We also record the small c expansions

$$B_1 = 1 + icz + O(c^2z^2), \tag{3.76}$$

$$B_2 = 1 - icz + O(c^2z^2), \tag{3.77}$$

$$C = z(1 - cz + O(c^2z^2)), \tag{3.78}$$

where it is clear that the solution reduces back to the undeformed one $B_1 = B_2 = 1$, $C = z$ when $c \to 0$.

The near horizon limit is given as before by taking

$$\tilde{x}^a := x^a/\alpha', \quad \text{and} \quad U := \rho/\alpha' \tag{3.79}$$

fixed in the $\alpha' \to 0$ limit. We also scale c such that

$$\tilde{c} := \alpha' c, \tag{3.80}$$

is fixed in the $\alpha' \to 0$ limit. We obtain the near horizon metric

$$\frac{ds^2}{\alpha'} = \frac{U^2}{\sqrt{\lambda}} dx_\mu^2 + \frac{\sqrt{\lambda}}{U^2}\left(\frac{1}{\tilde{A}}(d\tilde{x}_4^2 + d\tilde{x}_5^2) + \tilde{A}(d\tilde{x}_7^2 + d\tilde{x}_8^2) + \frac{1}{(1+\tilde{c}\tilde{z})^2(1+\tilde{c}^2\tilde{z}^2)} d\tilde{z}d\bar{\tilde{z}}\right),$$

where we have defined

$$\tilde{z} := z/\alpha', \tag{3.81}$$

The function $\tilde{A}$ is given by

$$\tilde{A} := \frac{1 - i\tilde{c}\tilde{z}}{1 + i\tilde{c}\tilde{z}} \tag{3.82}$$

and

$$U^2 = B_1\tilde{x}_i^2 + B_2\tilde{x}_m^2 + \frac{C}{\alpha'}\bar{\tilde{z}}, \tag{3.83}$$

where, with a slight abuse of notation, the coefficients B_1, B_2 and C are obtained from (3.71)–(3.75) by replacing cz with $\tilde{c}\tilde{z}$ everywhere. The RR 5-form is given by $F_0/\alpha'^2 = d(U^4)dx^{0123}/\lambda +$ dual, together with the constant components (2.37). This supergravity solution is the dual for the non-anticommutative deformed $\mathcal{N} = 4$ SYM with $\mathcal{N} = (1/2, 0)$ supersymmetry.

4. Some consequences of the correspondence

In this section we will consider the effect of the non-anticommutative deformation on the field theory, as predicted by the supergravity dual. We will concentrate on the duality with $\mathcal{N} = (1, 0)$ supersymmetry since in this case the supergravity background as constructed in section 3.1 is slightly simpler. In particular we will analyse the anomalous dimensions of the field theory operators.

As already noted in section 3.1 the metric is formally identical to the $AdS_5 \times S^5$ metric of the undeformed theory, subject to the replacement of z with $w = z/(1 + cz)$. However, there is one subtlety which we must first address. Since we have deformed the field theory in a non-Hermitian way, the dual supergravity solution has been modified by a complex deformation. The resulting effect is the replacement of z with w, but for $c \neq 0$, $w \neq z$. We therefore must be careful when interpreting this geometry. In particular, we need to identify the conformal boundary of the spacetime to relate bulk and boundary fields. Since we are viewing this theory as a deformation of the $\mathcal{N} = 4$ theory, we will use the standard notion of the boundary as $r \to \infty$ where $r^2 = x^i x_i + x^m x_m + |z|^2$. For $c \neq 0$ this differs from the complex quantity, $\rho^2 = x^i x_i + x^m x_m + w\bar{z}$ which naturally appears in many quantities. However, generically ρ diverges in the limit $r \to \infty$.

We will now consider the correspondence between bulk scalar fields and field theory operators. From the metric in equation (3.30) a scalar field K with mass m satisfies the Laplace equation which implies that

$$\left(\frac{\lambda}{\rho^4}\partial_\mu^2 + \partial_i^2 + \partial_m^2 + 4\partial_w\partial_{\overline{z}} - m^2\frac{\sqrt{\lambda}}{\rho^2}\right) K = 0 \tag{4.1}$$

As for the undeformed case, solutions of this equation which are independent of the “5-sphere” are given by

$$K = \frac{\xi^\Delta}{(x^\mu x_\mu + \xi^2)^\Delta} \tag{4.2}$$

where $\xi = 1/\rho$ and $\Delta = 2 + \sqrt{4 + m^2}$. We will now see that, despite the distinction between ρ and r, these states are dual to field theory operators with scaling dimension Δ. Therefore there is a class of field theory operators whose spectrum is not deformed. Note however that there are two possible ways in which the spectrum of operators can be deformed. There are the other solutions to the ten-dimensional Laplace equation which have a dependence on the “5-sphere”. Since this is deformed, the resulting spectrum of 5-dimensional masses will be changed. Also, the ten-dimensional spectrum of the full string theory is likely to depend on the deformation, giving a dependence of m^2 on c for string theory states.

We will now use the above solution $K(\xi, x^\mu)$ to give the 5-dimensional bulk to boundary propagator for these scalars and calculate the two-point function of the dual field theory operators using standard techniques. So, a boundary field $\phi_0(x^\mu)$ is a source for the bulk field configuration

$$\phi(\xi, x^\mu) = \int \mathrm{d}^4x' \frac{\xi^\Delta}{(|x-x'|^2+\xi^2)^\Delta}\phi_0(x') \tag{4.3}$$

To calculate the two-point function we need to consider the dependence of the action for the bulk scalar field on its "boundary values" $\phi_0(x)$. This is

$$\begin{aligned} I &= \int \mathrm{d}^4x\mathrm{d}r\mathrm{d}^5\Omega\frac{1}{2}\sqrt{g}\left(\partial_M\phi\partial_N\phi g^{MN}+m^2\phi^2\right) &(4.4)\\ &= \frac{1}{2}\int \mathrm{d}^4x\mathrm{d}^5\Omega\left(\sqrt{g}\phi g^{rN}\partial_N\phi\right)_{r\to\infty} &(4.5)\end{aligned}$$

where we have used integration by parts and the equation of motion for ϕ to perform the integral over r.

We can now use the above relation between ϕ and ϕ_0, together with the following standard polar parametrisation of the coordinates

$$\begin{aligned} x^6+ix^9 &= r\cos\alpha\mathrm{e}^{i\phi_1} &(4.6)\\ x^4+ix^5 &= r\sin\alpha\cos\theta\mathrm{e}^{i\phi_2} &(4.7)\\ x^7+ix^8 &= r\sin\alpha\sin\theta\mathrm{e}^{i\phi_3} &(4.8)\end{aligned}$$

to write the action explicitly in terms of the boundary sources ϕ_0. Note that for fixed angles, $\xi\to 0$ as $r\to\infty$. Explicitly, we find

$$\begin{aligned} \sqrt{g} &= r^5\xi^2H_2\sin^3\alpha\cos\alpha\sin\theta\cos\theta &(4.9)\\ \phi(\xi\to 0, x) &= \xi^{4-\Delta}\phi_0(x) &(4.10)\\ g^{rN}\partial_N\phi &= -\frac{\Delta}{H_2^2}\left((1+(H_2^2-1)\sin^2\alpha)\partial_r\rho+\frac{1}{r}(H_2^2-1)\sin\alpha\cos\alpha\partial_\alpha\rho\right)\times\\ &\quad\times\int\mathrm{d}^4x'\frac{\phi_0(x')}{|x-x'|^{2\Delta}} &(4.11)\end{aligned}$$

where in the last result we have kept only the leading order terms as $\xi\to 0$. Putting everything together we find the expression for the two-point function of the operator dual to the scalar field

$$\begin{aligned} \langle\mathcal{O}(x)\mathcal{O}(x')\rangle &= \frac{\delta I}{\delta\phi_0(x)\delta\phi_0(x')}\\ &= -\frac{\Delta}{2}\frac{1}{|x-x'|^{2\Delta}}\int\mathrm{d}^5\Omega\sin^3\alpha\cos\alpha\sin\theta\cos\theta\frac{r^5}{\rho^5H_2}\times\\ &\quad\times\left((1+(H_2^2-1)\sin^2\alpha)\partial_r\rho+\frac{1}{r}(H_2^2-1)\sin\alpha\cos\alpha\partial_\alpha\rho\right) &(4.12)\\ &= \frac{C}{|x-x'|^{2\Delta}} &(4.13)\end{aligned}$$

where the constant C is given by the integral in the previous line which is to be evaluated in the limit $r \to \infty$. Actually evaluating the integral is not straightforward since the large r behaviour is different for $\alpha = 0$ and $\alpha \neq 0$, as can easily be seen by considering the explicit expression for $\rho^2 = r^2 \sin^2 \alpha + r^2 \cos^2 \alpha/(1 + cr \cos \alpha e^{i\phi_1})$. Nevertheless the final result clearly indicates that the anomalous dimension of the operator $\mathcal{O}$ is given by Δ. Hence, for this class of operator, the only corrections to this dimension, as compared to the undeformed theory, can come from the possible dependence of the bulk mass m on the deformation parameter c.

Similar results will follow for field theory operators dual to other bulk fields. Due to the nature of the deformation, we expect that the spectrum of BPS states is simply a subset of the BPS states in the undeformed geometry. This would then correspond to the prediction that the scaling dimensions of the chiral operators in the field theory are the same as in the $\mathcal{N} = 4$ theory, but that the rest of the theory will be deformed.

5. Discussions

In this paper we have constructed the supergravity duals for the non-anticommutative deformed $\mathcal{N} = 4$ supersymmetric Yang-Mills theory with $\mathcal{N} = (1, 0)$ and $\mathcal{N} = (1/2, 0)$ supersymmetries. The supergravity solution consists of a metric which is a complex deformation of the $AdS_5 \times S^5$ metric, and a RR 5-form fields with complex constant components. The fact that the metric is non-dilatonic suggests that the field theory coupling is not renormalized. It will be interesting to check this explicitly.

Deformed by non-anticommutativity of the fermionic components of the superspace, the non-anticommutative field theory breaks supersymmetry in a novel non-traditional way. Nevertheless it preserves many remarkable properties of the usual supersymmetric field theories. It will be interesting to analyse and understand more this kind of supersymmetric breaking from the supergravity point of view.

The supergravity background dual to the non-anticommutative gauge theory is complex. The imaginary nature of the RR 5-form is easy to understand and is a direct consequence of solving the self-duality condition in Euclidean space. The imaginary nature of the metric is more obscure. Although we have demonstrated that one can nevertheless extract physical information such as the dimensions of operators in a more or less the standard way using the bulk-to-boundary approach, it will be good to have a deeper understanding on the imaginary nature of the metric. We recall that the complexity of the metric (and the flux background) is a direct reflection of the fact that the non-anticommutative field theory is non-Hermitian. With an analysis which is based on a reduction to the quantum mechanics, it has been suggested [40] that non-anticommutative theory is unitary in a more general sense [41]. This suggests something similar in the dual supergravity description. As a first step, it is natural to try to find the corresponding phenomena in the supergravity side when a similar reduction of degree of freedoms is performed. In the mini-superspace approximation, one can work out the canonical Hamiltonian. Due to the presence of complex components in the metric, the Hamiltonian is not real. We conjecture that the Hamiltonian is pseudo-real in a sense similar to its field theory counterpart. This

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would provide a physical understanding of the nature of the complexity of the supergravity background. We leave this interesting question for future analysis.

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A. Notation and convention

We denote by $\mu = 0, 1, 2, 3$ the directions in the 4-dimensional Euclidean worldvolume of the $D3$-branes, $a = 4, 5, 6, 7, 8, 9$ the transverse directions, and $\alpha, \dot{\alpha} = 1, 2$ the spinor indices. We follow the notation of [34]. In particular, spinor indices are raised and lowered by the ε-tensor with the convention $\varepsilon^{12} = -\varepsilon_{12} = 1$.

Decomposing SO(10) into SO(4) $\times$ SO(6), the ten-dimensional gamma matrices are given by

$$\Gamma^{\mu}_{(10)} = \gamma^{\mu} \otimes 1, \quad \Gamma^{a}_{(10)} = \gamma^{5} \otimes \Gamma^{a}, \tag{A.1}$$

where

$$\gamma^{\mu} = \begin{pmatrix} 0 & \sigma^{\mu} \\ \bar{\sigma}^{\mu} & 0 \end{pmatrix}, \quad \Gamma^{a} = \begin{pmatrix} 0 & \Sigma^{a} \\ \overline{\Sigma}^{a} & 0 \end{pmatrix}. \tag{A.2}$$

Here the matrices $(\sigma^{\mu})_{\alpha\dot{\beta}}$ and $(\bar{\sigma}^{\mu})^{\dot{\alpha}\beta}$ are given by

$$\begin{aligned} \sigma_{\mu} &= (i\tau^{1}, i\tau^{2}, i\tau^{3}, 1), \\ \bar{\sigma}_{\mu} &= (-i\tau^{1}, -i\tau^{2}, -i\tau^{3}, 1), \end{aligned} \tag{A.3}$$

where τ^{i} $(i = 1, 2, 3)$ are the Pauli matrices. They satisfy the Clifford algebra

$$\sigma_{\mu}\bar{\sigma}_{\nu} + \sigma_{\nu}\bar{\sigma}_{\mu} = 2\delta_{\mu\nu}\mathbf{1}. \tag{A.4}$$

The Lorentz generators are defined by

$$\sigma^{\mu\nu} = \frac{1}{4}(\sigma^{\mu}\bar{\sigma}^{\nu} - \sigma^{\nu}\bar{\sigma}^{\mu}), \quad \bar{\sigma}_{\mu\nu} = \frac{1}{4}(\bar{\sigma}_{\mu}\sigma_{\nu} - \bar{\sigma}_{\nu}\sigma_{\mu}). \tag{A.5}$$

The matrices $(\sigma_{\mu\nu})_{\alpha\beta}$, $(\tilde{\sigma}_{\mu\nu})^{\dot{\alpha}\dot{\beta}}$ are symmetric in the spinor indices. Moreover $\sigma_{\mu\nu}$ is self-dual and $\tilde{\sigma}_{\mu\nu}$ is anti self-dual with respect to the μ, ν indices.

The gamma matrices for six-dimensional part are given by

$$\begin{aligned} \Sigma^{a} &= \left(\eta^{3}, -i\bar{\eta}^{3}, \eta^{2}, -i\bar{\eta}^{2}, \eta^{1}, i\bar{\eta}^{1}\right), \\ \bar{\Sigma}^{a} &= (-\eta^{3}, -i\bar{\eta}^{3}, -\eta^{2}, -i\bar{\eta}^{2}, -\eta^{1}, i\bar{\eta}^{1}), \end{aligned} \tag{A.6}$$

where $a = 4, \ldots, 9$. $\eta^{a}_{\mu\nu}$ and $\bar{\eta}^{a}_{\mu\nu}$ are the 't Hooft symbols, which are defined by

$$\sigma_{\mu\nu} = \frac{i}{2}\eta^{a}_{\mu\nu}\tau^{a}, \quad \bar{\sigma}_{\mu\nu} = \frac{i}{2}\bar{\eta}^{a}_{\mu\nu}\tau^{a}. \tag{A.7}$$

The matrices (A.6) satisfy the Clifford algebra

$$(\Sigma^a)^{AB}(\bar{\Sigma}^b)_{BC} + (\Sigma^b)^{AB}(\bar{\Sigma}^a)_{BC} = 2\delta^{ab}\delta^A_C. \tag{A.8}$$

The charge conjugation matrix is block diagonal in this basis

$$\mathcal{C} = \mathcal{C}_{(4)} \otimes \mathcal{C}_{(6)}, \tag{A.9}$$

where

$$\mathcal{C}_{(4)} = \begin{pmatrix} -\epsilon^{\alpha\beta} & 0 \\ 0 & -\epsilon_{\dot{\alpha}\dot{\beta}} \end{pmatrix}, \quad \mathcal{C}_{(6)} = \begin{pmatrix} 0 & -i\delta_A{}^B \\ -i\delta_B{}^A & 0 \end{pmatrix}. \tag{A.10}$$

References

[1] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [*Int. J. Theor. Phys.* **38** (1999) 1113] [hep-th/9711200].

[2] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109].

[3] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].

[4] O. Aharony, S.S. Gubser, J.M. Maldacena, H. Ooguri and Y. Oz, *Large-N field theories, string theory and gravity*, *Phys. Rept.* **323** (2000) 183 [hep-th/9905111].

[5] G. 't Hooft, *Dimensional reduction in quantum gravity*, gr-qc/9310026.

[6] L. Susskind, *The World as a hologram*, *J. Math. Phys.* **36** (1995) 6377 [hep-th/9409089].

[7] J.M. Maldacena and J.G. Russo, *Large-N limit of non-commutative gauge theories*, *JHEP* **09** (1999) 025 [hep-th/9908134].

[8] A. Hashimoto and N. Itzhaki, *Non-commutative Yang-Mills and the AdS/CFT correspondence*, *Phys. Lett.* **B 465** (1999) 142 [hep-th/9907166].

[9] O. Lunin and J.M. Maldacena, *Deforming field theories with* $\mathrm{U}(1) \times \mathrm{U}(1)$ *global symmetry and their gravity duals*, *JHEP* **05** (2005) 033 [hep-th/0502086].

[10] C.-S. Chu and V.V. Khoze, *String theory dual of the beta-deformed gauge theory*, *JHEP* **07** (2006) 011 [hep-th/0603207].

[11] N. Dorey, T.J. Hollowood and S.P. Kumar, *S-duality of the Leigh-Strassler deformation via matrix models*, *JHEP* **12** (2002) 003 [hep-th/0210239].

[12] N. Seiberg, *Noncommutative superspace,* $N = 1/2$ *supersymmetry, field theory and string theory*, *JHEP* **06** (2003) 010 [hep-th/0305248].

[13] S. Ferrara and M.A. Lledo, *Some aspects of deformations of supersymmetric field theories*, *JHEP* **05** (2000) 008 [hep-th/0002084];
D. Klemm, S. Penati and L. Tamassia, *Non(anti)commutative superspace*, *Class. and Quant. Grav.* **20** (2003) 2905 [hep-th/0104190].

[14] C. Saemann and M. Wolf, *Constraint and super Yang-Mills equations on the deformed superspace* $R(h)(4|16)$, *JHEP* **03** (2004) 048 [hep-th/0401147].

JHEP05(2008)029

[15] M.T. Grisaru, S. Penati and A. Romagnoni, *Two-loop renormalization for nonanticommutative $N = 1/2$ supersymmetric WZ model*, *JHEP* **08** (2003) 003 [hep-th/0307099];
R. Britto and B. Feng, *$N = 1/2$ Wess-Zumino model is renormalizable*, *Phys. Rev. Lett.* **91** (2003) 201601 [hep-th/0307165];
A. Romagnoni, *Renormalizability of $N = 1/2$ Wess-Zumino model in superspace*, *JHEP* **10** (2003) 016 [hep-th/0307209];
O. Lunin and S.-J. Rey, *Renormalizability of non(anti)commutative gauge theories with $N = 1/2$ supersymmetry*, *JHEP* **09** (2003) 045 [hep-th/0307275];
M. Alishahiha, A. Ghodsi and N. Sadooghi, *One-loop perturbative corrections to non(anti)commutativity parameter of $N = 1/2$ supersymmetric $\mathrm{U}(N)$ gauge theory*, *Nucl. Phys.* **B 691** (2004) 111 [hep-th/0309037];
D. Berenstein and S.-J. Rey, *Wilsonian proof for renormalizability of $N = 1/2$ supersymmetric field theories*, *Phys. Rev.* **D 68** (2003) 121701 [hep-th/0308049].

[16] I. Jack, D.R.T. Jones and L.A. Worthy, *One-loop renormalisation of $N = 1/2$ supersymmetric gauge theory*, *Phys. Lett.* **B 611** (2005) 199 [hep-th/0412009]; *One-loop renormalisation of general $N = 1/2$ supersymmetric gauge theory*, *Phys. Rev.* **D 72** (2005) 065002 [hep-th/0505248];
M.T. Grisaru, S. Penati and A. Romagnoni, *Non(anti)commutative SYM theory: renormalization in superspace*, *JHEP* **02** (2006) 043 [hep-th/0510175].

[17] A. Imaanpur, *On instantons and zero modes of $N = 1/2$ SYM theory*, *JHEP* **09** (2003) 077 [hep-th/0308171]; *Comments on gluino condensates in $N = 1/2$ SYM theory*, *JHEP* **12** (2003) 009 [hep-th/0311137];
P.A. Grassi, R. Ricci and D. Robles-Llana, *Instanton calculations for $N = 1/2$ super Yang-Mills theory*, *JHEP* **07** (2004) 065 [hep-th/0311155];
R. Britto, B. Feng, O. Lunin and S.-J. Rey, *$\mathrm{U}(N)$ instantons on $N = 1/2$ superspace: exact solution and geometry of moduli space*, *Phys. Rev.* **D 69** (2004) 126004 [hep-th/0311275];
C.-S. Chu and T. Inami, *Konishi anomaly and central extension in $N = 1/2$ supersymmetry*, *Nucl. Phys.* **B 725** (2005) 327 [hep-th/0505141].

[18] H. Ooguri and C. Vafa, *The C-deformation of gluino and non-planar diagrams*, *Adv. Theor. Math. Phys.* **7** (2003) 53 [hep-th/0302109]; *Gravity induced C-deformation*, *Adv. Theor. Math. Phys.* **7** (2004) 405 [hep-th/0303063].

[19] J. de Boer, P.A. Grassi and P. van Nieuwenhuizen, *Non-commutative superspace from string theory*, *Phys. Lett.* **B 574** (2003) 98 [hep-th/0302078].

[20] N. Berkovits and N. Seiberg, *Superstrings in graviphoton background and $N = 1/2 + 3/2$ supersymmetry*, *JHEP* **07** (2003) 010 [hep-th/0306226].

[21] E. Ivanov, O. Lechtenfeld and B. Zupnik, *Nilpotent deformations of $N = 2$ superspace*, *JHEP* **02** (2004) 012 [hep-th/0308012].

[22] S. Ferrara and E. Sokatchev, *Non-anticommutative $N = 2$ super-Yang-Mills theory with singlet deformation*, *Phys. Lett.* **B 579** (2004) 226 [hep-th/0308021].

[23] S. Ferrara, E. Ivanov, O. Lechtenfeld, E. Sokatchev and B. Zupnik, *Non-anticommutative chiral singlet deformation of $N = (1, 1)$ gauge theory*, *Nucl. Phys.* **B 704** (2005) 154 [hep-th/0405049].

[24] K. Ito and S. Sasaki, *Non(anti)commutative $N = 2$ supersymmetric gauge theory from superstrings in the graviphoton background*, *JHEP* **11** (2006) 004 [hep-th/0608143].

[25] K. Ito, Y. Kobayashi and S. Sasaki, *Deformation of $N = 4$ super Yang-Mills theory in graviphoton background*, *JHEP* **04** (2007) 011 [hep-th/0612267].

[26] T. Araki, K. Ito and A. Ohtsuka, *Supersymmetric gauge theories on noncommutative superspace*, *Phys. Lett.* **B 573** (2003) 209 [hep-th/0307076].

[27] T. Araki, K. Ito and A. Ohtsuka, *$N = 2$ supersymmetric* U(1) *gauge theory in noncommutative harmonic superspace*, *JHEP* **01** (2004) 046 [hep-th/0401012].

[28] T. Araki and K. Ito, *Singlet deformation and non(anti)commutative $N = 2$ supersymmetric* U(1) *gauge theory*, *Phys. Lett.* **B 595** (2004) 513 [hep-th/0404250].

[29] E. Ivanov, O. Lechtenfeld and B. Zupnik, *Non-anticommutative deformation of $N = (1, 1)$ hypermultiplets*, *Nucl. Phys.* **B 707** (2005) 69 [hep-th/0408146].

[30] T. Araki, K. Ito and A. Ohtsuka, *Deformed supersymmetry in non(anti)commutative $N = 2$ supersymmetric* U(1) *gauge theory*, *Phys. Lett.* **B 606** (2005) 202 [hep-th/0410203].

[31] T. Araki, K. Ito and A. Ohtsuka, *Non(anti)commutative $N = (1, 1/2)$ supersymmetric* U(1) *gauge theory*, *JHEP* **05** (2005) 074 [hep-th/0503224].

[32] A. De Castro, E. Ivanov, O. Lechtenfeld and L. Quevedo, *Non-singlet Q-deformation of the $N = (1, 1)$ gauge multiplet in harmonic superspace*, *Nucl. Phys.* **B 747** (2006) 1 [hep-th/0510013];
A. De Castro and L. Quevedo, *Non-singlet Q-deformed $N = (1, 0)$ and $N = (1, 1/2)$* U(1) *actions*, *Phys. Lett.* **B 639** (2006) 117 [hep-th/0605187].

[33] M. Billó, M. Frau, I. Pesando and A. Lerda, *$N = 1/2$ gauge theory and its instanton moduli space from open strings in RR background*, *JHEP* **05** (2004) 023 [hep-th/0402160].

[34] J. Wess and J. Bagger, *Supersymmetry and supergravity*, Princeton Univeristy Press, Princeton U.S.A. (1992).

[35] A. Galperin, E. Ivanov, S. Kalitsyn, V. Ogievetsky and E. Sokatchev, *Unconstrained $N = 2$ matter, Yang-Mills and supergravity theories in harmonic superspace*, *Class. and Quant. Grav.* **1** (1984) 469.

[36] A.S. Galperin, E.A. Ivanov, V.I. Ogievetsky and E.S. Sokatchev, *Harmonic superspace* Cambridge University Press, Cambridge U.K. (2001).

[37] B.M. Zupnik, *The action of the supersymmetric $N = 2$ gauge theory in harmonic superspace*, *Phys. Lett.* **B 183** (1987) 175.

[38] R. Abbaspur and A. Imaanpur, *Nonanticommutative deformation of $N = 4$ SYM theory: the Myers effect and vacuum states*, *JHEP* **01** (2006) 017 [hep-th/0509220];
A. Imaanpur, *Supersymmetric D3-branes in five-form flux*, *JHEP* **03** (2005) 030 [hep-th/0501167].

[39] A.A. Tseytlin, *Composite BPS configurations of p-branes in 10 and 11 dimensions*, *Class. and Quant. Grav.* **14** (1997) 2085 [hep-th/9702163];
H. Lü and C.N. Pope, *Interacting intersections*, *Int. J. Mod. Phys.* **A 13** (1998) 4425 [hep-th/9710155];
D.J. Smith, *Intersecting brane solutions in string and M-theory*, *Class. and Quant. Grav.* **20** (2003) R233 [hep-th/0210157].

[40] E.A. Ivanov and A.V. Smilga, *Cryptoreality of nonanticommutative Hamiltonians*, *JHEP* **07** (2007) 036 [hep-th/0703038];
A.V. Smilga, *Physics of crypto-Hermitian and/or cryptosupersymmetric field theories*, *Phys. Rev.* **D 77** (2008) 061701 [arXiv:0710.5662].

[41] C.M. Bender and S. Boettcher, *Real spectra in non-hermitian hamiltonians having PT symmetry*, *Phys. Rev. Lett.* **80** (1998) 5243 [physics/9712001];
C.M. Bender, S. Boettcher and P. Meisinger, *PT-symmetric quantum mechanics*, *J. Math. Phys.* **40** (1999) 2201 [quant-ph/9809072].